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Bubbles slipping along a crenelated wall

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Abstract – We describe the ascent dynamics of air bubbles squeezed against a rigid crenelated plate immersed in a liquid bath. Depending on its shape and the inclination angle, the bubbles can remain trapped in the crenelations, or slip against the wall. In particular, the yield force separating the two regimes increases with the height of the crenels. In the second regime, the slipping motion is characterized by a capillary number which barely depends on the crenel geometry (wavelength and amplitude), at large enough angles. We finally discuss the regime of intermediate angles, and interpret the measured oscillations in bubble velocities through the existence of a (spatially) periodic force. These results provide information at the bubble scale for processes at play in the slip of foams and emulsions against rough walls.

Introduction. – Foams and emulsions are similar complex fluids encountered in numerous applications, including cosmetics, food industry, or oil recovery. They are both compact assemblies of soft deformable structures (bubbles or droplets) in a liquid matrix [1–3]. Macroscopically, their rheology is also similar as they are both yield stress fluids, whose elasticity arises from the interfacial energy of the bubbles or droplets. In addition, like other yield stress fluids, they exhibit wall slip along rigid walls, especially smooth ones [4–6], since the bubbles or droplets move at a different velocity than the wall. More precisely, the corresponding velocity jump, defined as slip velocity $V$, increases as a power-law of the shear stress at the wall, with an exponent measured between 0.5 and 1, and which depends on the system (emulsion or foam), on the dispersed phase volume fraction, on the chemistry of the solid surface and, for foams, on the mobility of the surfactants used to stabilize the liquid-gas interfaces [5–11].

From a practical point of view, while it may be interesting to enhance such slip to require less energy for fluid motion, wall slip needs to be avoided for accurate rheological measurements. A common strategy for this is to roughen the solid surface to promote adhesion —in the sense of no-slip—, with a roughness length scale comparable to the microstructure (bubble or droplet) size [4,5]. Beyond this empirical observation, Mansard et al. [11] have used microfluidic devices to investigate the influence of the roughness height on the slip velocity of concentrated emulsions. However, beside this pioneering work, an experimental characterization of the role of roughness on the transition between adhesion and slip in foams and emulsions, as well as an understanding of the mechanisms at play, are lacking so far.

On the other hand, in the case of foams slipping against smooth walls, a better understanding of the dissipation processes has been gained from the study of single foam film, single bubble or bubble monolayer experiments [9,12–15]. In line with theoretical and numerical works [16–18], different friction regimes have been identified and understood, depending on the liquid region undergoing the dominant dissipation, which can either be the liquid film separating the bubble, the liquid channels of the foams (far from the films), or the so-called dynamic meniscus which joins the two former zones. Following this approach, we investigate in this letter the dynamics of a single bubble close to a model rough wall, i.e., decorated with crenelations, with the aim to get some insight of the role of wall roughness on the slip of yield stress fluids like foams or emulsions.

Experimental setup and procedure. – The experimental setup consists in an inclined plane immersed in a bath of viscous liquid, similar to the setup of Aussillous and Quéré [14]. In order to focus on the interaction between the bubble deformation and the substrate structure, we chose silicone oils of two different dynamic viscosities $\eta = 19 \text{ mPa}s$ and $\eta = 97 \text{ mPa}s$, of comparable densities $\rho \approx 960 \text{ kg/m}^3$ and surface tensions $\gamma \approx 21 \text{ mN/m}$. This
choice ensures that the liquid totally wets the solid surfaces and suppresses dynamic surface tension effects due to surfactants or surface contaminants in aqueous solutions.

The plane consists in an acrylic-based resin plate (thickness 1 cm, width 4 cm, length 9 cm) obtained by 3D printing. The lower face of the plate is decorated with crenelated grooves, of wavelength \( \lambda = 3 \) mm and three different amplitudes \( A = 0.15, 0.3 \) and 0.6 mm (fig. 1(a)). Due to the printing technique that we use, the corrugations are not perfect squares but have slightly inclined edges, with a slope which is typically 30°, 45° and 80° for \( A = 0.15, 0.3 \) and 0.6 mm, respectively. To discuss our results, two additional surfaces are studied: a smooth one (no crenellations) and a crenelated one with medium amplitude (\( A = 0.3 \) mm) and larger wavelength (\( \lambda = 6 \) mm). The inclination of the plane is varied between \( \alpha = 1^\circ \) and 21° and is measured thanks to a plumb line.

Air bubbles with various sizes are generated inside the bath by using a needle and a syringe. Under buoyancy forces, the bubbles are pressed up against the plane and we measure their diameter \( D \) parallel to the plate (fig. 1(a)). \( D \) is generally comprised between 1 and 12 mm, i.e., comparable to the wavelength \( \lambda \) of the crenellations. It is also close to the capillary length of silicone oil \( a = \sqrt{\gamma/\rho g} \approx 1.5 \) mm, which compares gravity to surface tension effects. Finally, the height of the bubble depends on \( D \): for bubbles with \( D \) smaller than \( 2a \), it is approximately \( D \), while for bubbles larger than \( 2a \), gravity flattens the bubble as seen in fig. 1(a), and the bubble height is approximately \( 2a \) [14]; it is thus always larger than the crenel height \( A \).

When contacting the plane, if the inclination angle is large enough, the bubble will rise and slide along the surface. Its motion is recorded using a camera (IDS UI) equipped with an objective (image resolution 12–15 pixels/mm), the acquisition frequency of which is fixed between 20 and 80 Hz. Image processing performed using ImageJ (Fiji) software allows us to track the positions of the front and rear of the bubbles (fig. 1). The typical temporal evolution of these positions is shown in fig. 1(b) (dashed and dotted lines). In addition, we calculate the average of these two quantities (thick black line in fig. 1(b)), which we further call the average position or simply the position. Oscillations corresponding to the crenel wavelength \( \lambda \) can be seen, and an average drift velocity \( V \), here close to 3 mm/s, is measured (fig. 1(b)). The average bubble diameter \( D \) is also estimated by subtracting the front and rear positions.

**Yield force for bubble rising.** – For a given inclination of the plane, we wonder whether the bubble will slip on the solid surface or remain stuck inside crenellations. For a fixed crenelation geometry \( (A = 0.3 \) mm and \( \lambda = 3 \) mm), our observations are summarized in the phase diagram \( (D, \alpha) \) shown in fig. 2(a). We first observe that bubbles will move (on distances larger than a few wavelengths) when they are large enough. Besides, for the same bubble diameter, the bubble will slide up only at large enough angles. This behaviour is simply due to the fact that the driving buoyancy force increases with both \( D \) and \( \alpha \): \( F = \rho g A \sin \alpha \), where \( \Omega \) denotes the bubble volume, which increases with the bubble diameter. In other words, the bubble slips if \( F \) is larger than a yield force \( F_y \). Note that a sufficient condition for sliding is when the inclination \( \alpha \) is larger than the largest slope of the crenel profile (between 30° and 80°), but this is never the case in our experiments where \( \alpha = 1^\circ \)–21°. Finally, this onset of motion is independent of the liquid viscosity \( \eta \) (different colors and symbols in fig. 2(a)); this is not surprising since we might expect the yield force to depend only on the (static) deformability of the bubble against the crenels, hence on surface tension, gravity, bubble size and crenel geometry, but not liquid viscosity.

From these data, we can deduce for each roughness geometry a yield diameter \( D_y \) as a function of the angle \( \alpha \), above which bubbles will slip along the rough surface, corresponding to the frontier between the two domains (slip or stick). Results for \( D_y(\alpha) \) are shown for three different crenel heights \( A \) (fig. 2(b)) and two different wavelengths \( \lambda \) (fig. 2(c)). We observe that the yielding diameter increases with \( A \): the higher the crenel, the larger the yield force, as intuitively expected. Besides, the influence of the crenel periodicity \( \lambda \) on the yielding behaviour is less
clear. In particular, for \( \lambda = 6 \text{ mm} \), a few data points (for \( \alpha \approx 4-6^\circ \)) lie above the \( \lambda = 3 \text{ mm} \) points, but the difference is not significant given the error bars (based on the diameter measurement error, and the distance between stick and slip data points on the phase diagram).

In order to interpret these data, we estimate the restoring force due to buoyancy which traps the bubble inside the crenelations, assuming for simplicity a perfect square shape for the crenels and neglecting the inclination of the step edges. We first evaluate the change in potential gravitational energy when a portion \( \varepsilon \) of the bubble touches the bottom of the crenels: \( \Delta E \sim \rho g \Omega A \varepsilon \), assuming \( \alpha \ll 1 \) and that the bubble height is much larger than \( A \). \( \varepsilon \) depends on the position \( x \) of the bubble. The restoring force due to gravity is then \( F_Y = (d[\Delta E]/dx)|_{x=0} = \rho g \Omega A \varepsilon /x \).

Let us first consider the case of bubbles larger than the capillary length \( (D \gg 2a) \) which are flattened by buoyancy forces, and larger than the crenel wavelength \( \lambda \). As the bubble moves by a distance \( \lambda/2 \), it leaves a crenelation, and the corresponding change in \( \varepsilon \) is \( \sim (\lambda/2)/D \), from which we get \( F_Y \sim \rho g \Omega A (\lambda/2D)/2(\lambda/2) \sim \rho g \Omega A D \).

On the other hand, if \( D \) is smaller than \( \lambda \) (but still larger than \( 2a \)), \( \varepsilon \) varies from 0 to 1 on a typical size \( D \), so that \( F_Y \sim \rho g \Omega A \times (1/D) \). In both cases, assuming that the bubble volume reads \( \Omega \sim D^2a \) [14], we expect \( F_Y \sim \rho g a AD \).

Finally, we consider the limit of bubbles smaller than both \( 2a \) and \( \lambda \). In this case, the bubble volume is \( \Omega \sim D^3 \). We now need to determine on which length scale (parallel to the average slope of the plane) the bubble “feels” a step of the crenel. For tiny bubbles which are perfectly spherical, the bubble will touch the step when its center is at a distance \( \ell_0 \sim \sqrt{AD} \) from the step. However, for intermediate sizes, the bubble are slightly flattened by gravity and there is a flat “contact” liquid film which separates the bubble from the solid plane (defined in fig. 1(a)); the diameter of this flat film scales as \( \ell \sim D^2/a \) [14,19]. Figure 2(b) shows that we observe yielding for \( D \geq 2 \text{ mm} \), so that we always have \( \ell > \ell_0 \) in experiments: \( \ell \) is thus the typical length scale on which the bubble profile is disturbed by the step. The typical restoring force is therefore \( F_Y \sim \rho g \Omega A/\ell \sim \rho g a AD \), as in the previous cases. As expected, we find that the higher the crenels, the larger the yield force. We also note that this expression is independent of the crenel wavelength \( \lambda \).

At the yielding point, \( F_Y \) balances the average buoyancy force \( \rho g a \sin \alpha \). This leads to \( D_Y \sim \sqrt{aA/\sin \alpha} \) for \( D_Y \ll 2a \) and \( D_Y \sim A/\sin \alpha \) for \( D_Y \gg 2a \). We thus plot in the inset of fig. 2(b) \( D_Y \) as a function of \( A/\sin \alpha \), in logarithmic scales. We find that this representation gathers all our data. Besides, a 1/2 power law is able to describe the variation of \( D_Y \) with \( \alpha \) over the whole diameter range, which is expected from our simple prediction only for bubbles smaller than \( 2a \). For \( D \gg 2a \), this suggests that the yielding force saturates when increasing \( D \), which is not predicted by our simple model: for a given diameter, the yielding angle is such that \( \sin \alpha \approx LA/D^2 \), with \( L \approx 5.7 \text{ mm} \), from which we can evaluate the yield force which balances the buoyancy force \( F_Y \sim \rho g D^2a \sin \alpha \sim \rho g AaL \). It is indeed independent of \( D \). Understanding this saturation and whether it occurs for bubble diameters larger than the capillary length \( a \) or the wavelength \( \lambda \) remains an open question, which would require further investigation with a larger range of roughness wavelength.
High-velocity regime. – We now turn to the flow regime, for which the bubble slips upwards at a constant velocity, for angles and bubble diameters above the yielding threshold characterized above. For bubbles creeping at velocity $V$ along a smooth plane, the friction force is determined by the coupling between viscous dissipation and the surface energy which resists the bubble deformation, characterized by the capillary number Ca = $\eta V/\gamma$: in particular, for bubbles larger than the capillarity length ($D > 2a$), the drag force is given by Bretherton law $F \sim \rho V D^2 a \sin \alpha$ yields $Ca \sim (D/a)^{3/2} \sin^{3/2} \alpha$ (for $D \gg a$). For a given inclination angle $\alpha$, and provided the capillary length $a$ is the same, Ca is thus expected to be an increasing function of only the bubble size.

Figure 3 shows the capillary number Ca (deduced from our velocity measurements), as a function of the bubble diameter $D$, at different inclination angles $\alpha$. The various datasets correspond to different roughness geometries and two different viscosities. The surfaces are either smooth or crenelated with a factor 4 in crenel height $A$ and a factor 2 in wavelength $\lambda$, while the ratio between the two viscosities is 5. The most important result, and one of the main results of the present letter, is clearly visible in figs. 3(a)–(d), which correspond to the largest angles ($\alpha \geq 7^\circ$) in our experiments: the measured capillary numbers appear as independent of the roughness characteristics for all the surfaces tested in our experiments.

In particular, this experimental fact contrasts with measurements of wall slip velocities in concentrated emulsions by Mansard et al. [11]. In this work, the authors investigated the influence of the depth of an etched pattern at the wall of a microfluidic channel. For depths between 1 and 3 microns, corresponding to $A/D$ in the range 0.14 to 0.4 (with $D \approx 7 \mu m$ the droplet diameter), they find a slip velocity smaller by a factor close to 3. For comparison, in our model bubble experiment, we measure the same slip velocities on rough and smooth surfaces, while the typical ratio $A/D$ varies between 0.025 and 0.1 (taking $D = 6 mm$). Note that if we compare $A$ to the bubble height $\approx 2a$ (valid for large bubbles [14]), the ratio is then in the range 0.05–0.2. For $D \approx 6 mm$, this result is found in a large range of inclination angles ($\alpha \geq 7–20^\circ$), corresponding to a driving force varying by a factor 2.6.

Fig. 3: (Color online) Capillary number $Ca = \eta V/\gamma$ as a function of the bubble diameter $D$, for all the crenelation geometries.

For each figure, the inclination angle $\alpha$ is fixed in the following ranges: (a) $\alpha = 20.1–21.3^\circ$, (b) $\alpha = 11.6–12.6^\circ$, (c) $\alpha = 9.1–10.9^\circ$, (d) $\alpha = 6.7–8.0^\circ$, (e) $\alpha = 3.9–5.8^\circ$ and (f) $\alpha = 2.5–3.0^\circ$. Each symbol and color correspond to a given crenelation design: smooth surface (black circles), $\lambda = 3$ mm and $A = 0.15$ mm (brown triangles), $A = 0.30$ mm (red squares) and $A = 0.6$ mm (light red diamonds), and $\lambda = 6$ mm and $A = 0.3$ mm (blue stars). Empty symbols correspond to $\eta = 19$ mPa s and solid symbols correspond to $\eta = 97$ mPa s. The solid lines show the fit $Ca = \beta(D/a)^{3/2}$ for $D \geq 2a$. (c) Inset: fitted coefficient $\beta$ as a function of $\sin^{3/2} \alpha$. The solid line is a linear fit with slope 0.093. This prefactor can be compared to the experimental results of Aussillous and Quéré [14]: if we assume that the flattened bubbles have an ellipsoidal shape with volume $\Omega = \pi D^2 a/3$, we find for their data a prefactor $0.34(1/4)^{3/4} \approx 0.12$. 

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One could argue that this difference arises from the change in hydrodynamic boundary conditions due to the change of inner fluid (gas for the single-bubble experiment vs. oil in the emulsion) and to the presence of surfactants in the emulsion. However, their major impact is to change the dissipation in the flat liquid films which separate the bubbles from the wall [18], whereas in the results of Mansard et al. [11], the drag force is roughly linear in velocity, suggesting that dissipation in the liquid films is not dominant.

More likely, this indicates that the correspondence between single bubble (resp. droplet) and foam (resp. emulsion) dynamics against a rough wall is not as straightforward as in the case of a smooth wall [6,14], and that a description of the whole bubble/droplet assembly is needed to describe the slip behavior of complex fluids like foams or emulsions.

In addition, we can check in figs. 3(c) and (d) that the effect of the viscosity $\eta$ on the velocity $V$ is reasonably taken into account by the capillary number. Besides, the data for $D > 2a$ can be well fitted by the prediction $Ca \sim (D \sin \alpha/a)^{3/2}$, as shown by the solid lines and the inset of fig. 3, although the prefactor is somewhat lower (by 25%) than the one found by Aussillous and Quéré [14] for a more viscous oil. This could be due to inertial effects which may be non negligible in the present experiment [21].

Finally, for a given $\alpha$, we verify that, on rough surfaces, there is no flow at low diameters $D < Dv$. This is more pronounced for the smallest angles (figs. 3(e), (f)), for which $Dv$ is larger, as discussed above. The absence of effect of the crenelations on the velocity is visible provided we are far enough from yielding, typically $D \gtrsim 1.5Dv$, corresponding to $F \gtrsim 2FY$ (since $F \propto D^2$ for bubbles larger than the capillary length [14]).

**Velocity oscillations in intermediate regimes.** – We finally discuss the temporal evolution of the bubble position and velocity. Observations of bubble trajectories suggest fluctuations of the bubble position, whose spatial wavelength is given by the crenel wavelength $\lambda$ (fig. 1(b)). These correspond to oscillations in the instantaneous bubble velocities as shown on fig. 4(a) for two different inclination angles. The instantaneous velocities are obtained by differentiating the measured position as a function of time. We observe that the relative amplitude of these oscillations increases when the angle is decreased, that is, closer to yielding. Besides, the oscillations are asymmetric: the time-average velocity is lower than the average of extreme values.

In order to qualitatively understand this behavior, we consider a minimal model for the bubble position $x$. We assume the bubble experiences a driving force $F$, a viscous friction $-C\dot{x}$ and a force which varies periodically in space $f \cos(2\pi x/\lambda)$, which models the influence of the roughness on the bubble dynamics. The amplitude $f$ of the force field corresponds to the yield force $FY$, and $F$ is the equivalent of the buoyancy force $\rho g h \sin \alpha$ in our experiment. Thus, for fixed bubble size and roughness, the ratio $F/f$ increases with the inclination angle $\alpha$.

Balancing the forces yields the bubble velocity as a function of its position $v = \dot{x} = [f \cos(2\pi x/\lambda) + F]/C$. The minimum (resp. maximal) velocity is $v_{\text{min}} = (F - f)/C$ (respectively, $v_{\text{max}} = (F + f)/C$) and the amplitude of the velocity oscillations reads $\Delta v = v_{\text{max}} - v_{\text{min}} = 2f/C$. The expression of $v$ can be integrated numerically to extract the temporal evolution of position and velocity. The result of the calculation is shown in fig. 4(b) (in dimensionless variables) and shows the same qualitative trend as the experiments, as the driving force $F$ is decreased towards $f$. Besides, the average velocity can be deduced:

$$\langle v \rangle_t = \frac{\lambda}{\int_0^\lambda dx/v(x)} \frac{2\pi(f/C)}{\int_0^{2\pi} dy/\left(\cos y + F/f\right)} = \sqrt{F^2 - f^2}/C.$$

In particular, we have $\langle v \rangle_t = \sqrt{v_{\text{min}}v_{\text{max}}} < (v_{\text{min}} + v_{\text{max}})/2$. We thus recover the asymmetry of the velocity oscillations visible in the experiments (fig. 4(a)). This simple model of periodic force thus captures the qualitative features of the single bubble dynamics; it could thus
be used as a microscopic ingredient for mimicking the flow of foams and emulsions close to more or less rough walls in simulations of jammed soft particles [22–25].

Finally, for a quantitative and comprehensive understanding of these oscillations, a complete description of the bubble interface dynamics would be necessary, which is beyond the scope of the present article; it is indeed visible in fig. 1(b) that the liquid gas interface oscillates more at the rear than at the front of the bubble.

**Conclusion.** – We have characterized the trapping and the motion of bubbles against an inclined crenelated wall immersed in a bath of viscous silicone oil. We show that the bubbles stick to the wall when they are small enough or for low inclination angles, indicating the existence of a yield force. This yield force increases with the crenel height as expected. Above the yield force, the bubbles slip at a velocity that appears independent of the crenelation geometry (crenel height and wavelength), provided the driving buoyancy force is large enough, compared to the yield force. For intermediate forces, the instantaneous velocities show oscillations, which can be qualitatively understood through a minimal model of a bubble undergoing a force varying periodically in space.

In particular, these results provide local ingredients (at the bubble scale) for modeling and simulations of soft-jammed complex fluids like foams or emulsions [22–25]. Such systems are known to exhibit wall slip, except when the solid boundaries are corrugated enough. The role of the roughness on this phenomenon remains scarcely explored [11,26]. However, as discussed above, the measured slip velocities in emulsions [11] differ from our results at the single-bubble level. Although direct comparisons of single bubble/droplet and foam/emulsion experiments (with similar physico-chemistry) would be necessary to delineate the influence of inner fluid viscosity and surfactant-laden interfaces, this suggests that taking into account collective effects—bubble-foam or droplet-emulsion interactions—in foams and emulsions will be necessary to achieve a complete understanding of the effect of boundaries on these complex flows.

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