Dense pedestrian crowds versus granular packings: An analogy of sorts

Alexandre NICOLAS

Institut Lumière Matière, CNRS and Université Claude Bernard Lyon 1, F-69622 Villeurbanne, France
ORCiD: 0000-0002-8953-3924
alexandre.nicolas@polytechnique.edu

Abstract. Analogies between the dynamics of pedestrian crowds and granular media have long been hinted at. They seem all the more promising as the crowd is (very) dense, in which case the mechanical constraints prohibiting overlaps might prevail over the decisional component of pedestrian dynamics. These analogies and their origins are probed in two distinct settings, (i) a flow through a narrow bottleneck and (ii) crossing of a static assembly by an intruder. Several quantitative similarities have been reported for the former setting and are discussed here, while setting (ii) reveals discrepancies in the response pattern, which are ascribed to the pedestrians' ability to perceive, anticipate and self-propel.

Keywords: pedestrian dynamics; crowd dynamics; granular media; bottleneck flow

Besides its obvious practical relevance, the study of pedestrian dynamics owes much of its interest to its singular positioning at the crossroads between social sciences and mechanics. This is reflected by a dichotomy of approaches.

On the one hand, statistical models inspired from economics, such as discrete-choice models[1], are often used when it comes to predicting distant destinations or routes chosen by pedestrians [1]. (Nevertheless, it should be mentioned that they have also been applied to stepping dynamics [2]).

On the other hand, mechanical approaches are generally favoured when the dynamics are investigated in finer detail, at a more 'microscopic' scale, in particular when strong interactions or hindrances are expected between pedestrians because of the crowd's density. These approaches are formally based on Newton's equation of motion (or variants thereof), supplemented with a term describing the (externally inferred) desired direction.

How the pedestrians' *decisions* and *choices* of motion in view of their surroudings should enter this mechanical framework remains however unclear, from a fundamental viewpoint. *In practice*, the two components (the 'cognitive' one and the purely mechanical one) are often combined in an *ad hoc* way. In the

[1] These models depend on an abstract utility function, which quantifies the attractiveness of a given destination, direction or trajectory.

famous social-force model [3], at the heart of various commercial software products, these components are simply put on an equal footing, in that the choices of motion (influenced by the other pedestrians and the built environment) are converted into 'social forces' and summed up with the mechanical forces in Newton's equation of motion. One is left with a mechanical problem, without further justification.

Mechanical descriptions are quite appealing, especially at high density. Indeed, when the crowd is very dense, one may even think that excluded-volume effects (precluding overlaps) restrict the possibilities of motion so much that they stifle the effect of decision-making, thus the singularities of pedestrians as compared to, say, grains of matter. To translate this idea into a reasoning on configuration space, most configurations are admissible for a sparse crowd, because randomly positioned pedestrians seldom overlap; in contrast, at very high density, volume exclusion reduces the available volume of configuration space; the prevalence of areas that are forbidden because of overlaps might even restrict the possible directions of evolution of the assembly in configuration space to such an extent that the former virtually govern its evolution (as if in a maze).

This begs the following question: Is a purely mechanical description satisfactory at very high density? Does the crowd then behave like a granular medium? We shall address this question in two distinct situations, a bottleneck setup in Section II and an intrusion experiment in Section III, after setting the theoretical framework in Section I.

This manuscript aims to provide a pedagogic discussion. Thus, most quantitative features are skimmed off; references to already published or forthcoming manuscripts will be provided for readers interested in quantitative aspects.

1 Theoretical framework

We start with a brief introduction to the theoretical framework in which the motion of pedestrians is to be handled, within classical mechanics. As a mechanical body, a pedestrian i obeys Newton's equation of motion,

$$m\ddot{\boldsymbol{r}}_i = \boldsymbol{F}_{\text{ground}\to i} + \boldsymbol{F}^{(\text{mech})}_{\text{wall}\to i} + \sum_j \boldsymbol{F}^{(\text{mech})}_{j\to i}.$$

Here, the *mechanical* forces possibly exerted by walls, $\boldsymbol{F}^{(\text{mech})}_{\text{wall}\to i}$, and other pedestrians j, $\boldsymbol{F}^{(\text{mech})}_{j\to i}$, in case of contact and pushes, have been taken into account, even though they seldom arise in practice. The ground force, $\boldsymbol{F}_{\text{ground}\to i}$, is composed of (i) a static (vertical) reaction force that would subsist with no deformation of the pedestrian's body (*passive case)*, and (ii) a force $\boldsymbol{F^{(p)}}_i$ that results from changes in the body shape (created by flexing leg muscles) and that propels the pedestrian. In the horizontal plane, one can thus write

$$m\ddot{\boldsymbol{r}}_i = \boldsymbol{F^{(p)}}_i + \boldsymbol{F}^{(\text{mech})}_{\text{wall}\to i} + \sum_j \boldsymbol{F}^{(\text{mech})}_{j\to i}.$$

Note that this self-propelling force $\boldsymbol{F^{(p)}}_i$ does not require any process akin to decision-making: Active particles such as vibration-driven 'hexbugs' [4, 5] are also propelled by $\boldsymbol{F^{(p)}}_i$. Entities that make decisions about their motion on the basis of their perceptions [6, 7] are nonetheless singular in that their (active) propelling force will be a function of *complex dependences on environmental cues, the history of past body shapes*, etc., generically denoted by '(...)'; see Fig. 1. Noting that the reaction to external cues cannot be instantaenous, it needs a finite time $\tau_\psi > 0$ to be processed (for humans, voluntary reactions take at least 0.1 s; for instance, the behavioural response to a complex visual stimulus occurs after typically $\tau_\psi = 0.4\,\mathrm{s}$ [8]), one can write

$$\begin{cases} m\ddot{\boldsymbol{r}}_i & = \boldsymbol{F^{(p)}}_i + \boldsymbol{F}^{(\mathrm{mech})}_{\mathrm{wall}\to i} + \sum_j \boldsymbol{F}^{(\mathrm{mech})}_{j\to i} \\ \tau_\psi \dot{\boldsymbol{F}}^{\boldsymbol{(p)}}{}_i & = f(\ldots) \end{cases} \quad (1)$$

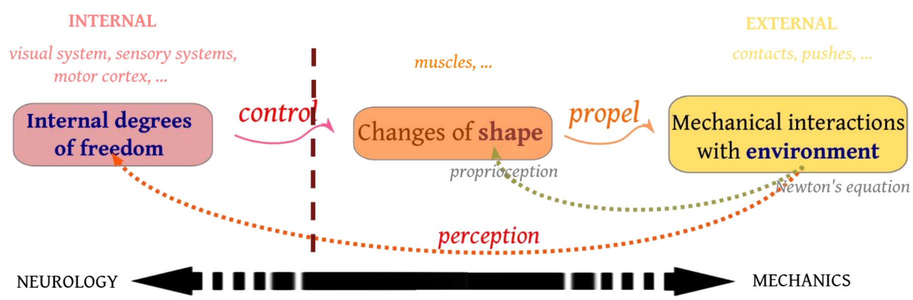

Fig. 1. Schematic representation of the interplay between external mechanical forces, the self-propelling force, and non-mechanical cues from the environment.

Of course, the foregoing derivation of the singularity of perceptual entities is overly simplified, because $\boldsymbol{F^{(p)}}_i$ may in fact also vary with the configuration of the crowd for non-perceptual entities, for instance if a magnetic self-propelled particle has its polarisation axis rotated via magnetic interactions with its neighbours. Nonetheless, it is interesting to note that, even in the case of such an elementary dependence of $\boldsymbol{F^{(p)}}_i$ on the environment, the collective behaviour of self-propelled particles may differ from that of their passive driven counterparts, notably in their self-organisation abilities [9]. A more accurate view on the topic is reserved for a forthcoming paper (Nicolas, *in preparation*).

In the following, we wonder whether the complex dependences of $f(\ldots)$ in Eq. 1 could not be overlooked at high densities, due to the prevalence of the mechanical constraints embodied by the $\boldsymbol{F}^{(\mathrm{mech})}$'s, which would thus liken the crowd's response to that of a granular medium.

2 Bottleneck flows

The foregoing question is first addressed in the context of competitive flows through a (narrow) bottleneck, insofar as this setting seems to severely constrain the possibilities of motion.

2.1 Common features

In the last few years, several similarities (which had long been hinted at in a tentative way) have been quantitatively demonstrated in experiments with granular materials flowing out of a vibrated hopper, on the one hand, and pedestrian crowds asked to rush through a narrow doorway, on the other hand [10]. These similarities include:

- exponential bursts of egresses in close succession (see [11] for a discussion about the universality of this feature)
- heavy tails in the distribution of time gaps τ between egresses in competitive escapes, which are at least reasonably well described by power laws: $p(\tau) \sim \tau^{-\alpha}$ [12]. Note that this expression entails a divergence of the mean time gap $\langle \tau \rangle$ when $\alpha < 2$, hence a dwindling flow rate as the system size tends to infinity.
- the possibility of a faster-is-slower effect in the evacuation, whereby tuning up the entities' drive to escape (*by tilting the hopper or prescribing a more competitive behaviour to the pedestrians*) may actually delay the evacuation due to longer clogs [12]. It should however be noted that, while the possibility of such an effect has been experimentally demonstrated with crowds, it requires high competitiveness and perhaps also aggressive pushes; for less competitive evacuations, a more trivial faster-is-faster effect is observed [13, 14]

Given that the flow rate J is given by $J = 1/\langle \tau \rangle$, the distribution $p(\tau)$ of time gaps τ is a central feature, which we now study in granular flows and then in their pedestrian counterparts.

2.2 Granular hopper flows

Long time gaps are caused by the formation of clogs, due to arches blocking the aperture of the vibrated hopper in two dimensions. The ultimate shattering of an existing arch, if it occurs, owes mostly to the vibration-induced destabilisation of the weakest link in the arch, i.e., of the grain that forms the widest angle with its neighbours in the arch [15]. Thus, we were led to study the evolution of the position of this grain under vibrations. Using non-trivial, but reasonable assumptions, we showed that this evolution can be likened to that of a Brownian particle in an energy trap, with an inverse temperature β associated with the acceleration Γ of the vibrations, $\beta \approx \frac{\gamma}{\Gamma^2}$, where γ is a drag coefficient [16]. It follows from this formal mapping that the lifetime of an arch is given by the Kramers' escape time

$$\tau \propto \exp\left(\beta\, E_b\right), \tag{2}$$

where E_b is the trap depth (or energy barrier). The values of E_b were inferred from ramp experiments, in which the intensity of vibrations was gradually increased until an existing arch broke; they were shown to follow a Weibull distribution

$$p(y) = e^{-y}\, \text{with}\, y \propto \sqrt{E_b}$$

Arch stabilities, quantified by E_b, are thus fairly narrowly distributed. But the bottomline is that this fairly narrow disorder is amplified by the exponential dependence on E_b in Kramers' escape time, eq. 2. Were energy barriers exponentially distributed (as they are in the Soft Glassy Rheology model [17]), this would lead to a power-law distribution of escape times τ. Here, the disorder in the arch stabilities is slightly larger, therefore the distribution of arch lifetimes decays even more slowly than a power law as $\tau \to \infty$. The results of the foregoing minimal model attain semi-quantitative agreement with experimental data [16]. Importantly, they lead to the prediction that the mean time gap $\langle\tau\rangle$ may diverge irrespective of the vibration intensity (in the limit of moderate shaking), which casts doubt on the ability of gentle vibrations to restore the hopper flow persistently.

2.3 Pedestrian flows through a bottleneck

Turning to pedestrians, temporary clogs are also observed in competitive pedestrian flows through a bottleneck, due to pedestrian 'arches' blocking the doorway. Our initial observation was that cellular automata (CA) provide a computationally very efficient way to simulate them, e.g., to test evacuation scenarios, but that their predictions are seldom validated against experimental measurements, in particular detailed statistics about the time series of egresses [12]. As a matter of fact, it seemed to us that none of the CA that we surveyed in the literature was able to describe the seemingly heavy-tailed distribution of time gaps between egresses reported for highly competitive evacuations [18]. This feature turned out to be difficult to replicate. Indeed, the regular grid typically used in CA precludes geometric disorder in the pedestrian arches, whereas such disorder was central to the replication of heavy tails in granular flows (see the previous section).

It goes without saying that the specific mechanics governing the clogging dynamics differ between pedestrians and grains. However, the broad insight gained from the granular case helped us devise a CA that was finally able to reproduce experimental results on controlled evacuations semi-quantitatively. To this end, after realising (i) the importance of having some intrinsic disorder in the model (and not just random fluctuations) and (ii) the constraints affecting the geometry in a CA, we decided to introduce disorder in the agents' behaviours, more precisely in the impatience displayed by agents close to the exit (the so called 'friction'), which controls the number of conflicting endeavours to escape simultaneously. This enabled us to capture the desired experimental features [18].

All in all, despite the discrepancies in the physical origins of their motion, (competitive) pedestrians and grains flowing through a narrow bottleneck exhibit striking similarities, which hints that basic mechanisms control the dynamics in this setup, in particular the joint presence of a driving 'force' towards the escape and paramount volume exclusion effects (included in $\boldsymbol{F}_{j\to i}^{(\mathrm{mech})}$ in Eq. 1) [2].

3 Crossing by an intruder

To test if the similarities observed in bottleneck flows extend to other setups, we now move on to crossing tests of a static (pedestrian or granular) two-dimensional assembly by an intruder.

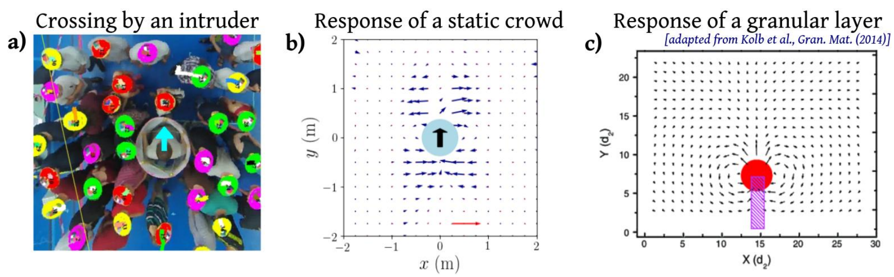

Fig. 2. Response to the crossing of an intruder in a static crowd (a-b) and a granular layer (c). Picture (a) is a snapshot taken during a controlled experiment, while panels (b) and (c) are average flow fields in the co-moving frame of the intruder. The red arrow in (b) represents a velocity of 30 cm/s. Adapted from Ref. [20, 21].

3.1 Granular mono-layer

This kind of test is fairly classical for granular media. In particular, Cixous, Kolb and colleagues studied the passage of a circular intruder in a mono-layer of granular disks below jamming [21] (also see the related works by Séguin et al. [22]). They distinguished different regimes, but in the regime most relevant for the upcoming comparison they observed an average flow field of the form displayed in Fig. 2c. It features radial velocities on the fore-side of the intruder, with recirculation eddies on the side. Furthermore, the perturbation is short-ranged and decays exponentially in space (contrary to what happens in a viscous fluid). It should also be stated that a depleted zone appears in the wake of the cylinder, with a size that decreases with increasing packing fraction.

[2] Note that these similarities may perhaps not extend to the effect of placing an obstacle in front of the door [19].

3.2 Static pedestrian crowd

To allow comparison, controlled experiments were performed in Orsay (France) and Bariloche (Argentina), in which a cylinder of around 70 cm in diameter moved linearly through a group of static participants (see Fig. 2a). The group's density was varied from $1-2\,\mathrm{ped/m^2}$ to $6-7\,\mathrm{ped/m^2}$ and its orientation was also varied; for further experimental details, refer to [20].

In short, when the participants were asked to behave casually, as if they were on an underground platform, and were facing the incoming intruder, the crowd's response consisted of strictly transverse moves directed outwards downstream from the obstacle and inwards in its wake, as shown in Fig. 2b, with only little sensitivity to the crowd's density. This is in stark contrast with the granular response of Fig. 2c. On the other hand, similarly to the granular case, the perturbation was short-ranged and left a pedestrian-free zone in the wake of the obstacle, which was refilled after the passage, via the inwards moves mentioned just above.

Only when the participants were asked to refrain from anticipating the intruder's passage *and* had to turn their back to the intruder (which thus arrived from behind), only then did the crowd's response mirror the granular one somewhat more closely, with displacements aligned approximately radially on the fore-side of the intruder (*data not shown here*).

3.3 Pedestrian specificities

As in the bottleneck setup, but to a much lesser extent here, some similarities were noticed in the pedestrian and granular responses, which consist of a short-ranged perturbation field (possibly due to the discreteness of the systems) and a depleted zone in the intruder's wake. In contrast, the observed discrepancies in the displacement patern are ascribed to the following two salient specificities of pedestrians, as compared to grains, which were also highlighted in the schematic diagram of Fig. 1.

Firstly, self-propulsion grants pedestrians the possibility to move along a direction that is not aligned with the mechanical force applied by their surroundings (i.e., the intruder and the crowd). Secondly, the internal 'decision-making' process informed by the perception of the environment allows pedestrians to anticipate the passage of the intruder, hence start moving before contact and elect a transverse move, rather than an axial one (so as to dodge out of the intruder's way, instead of delaying the collision).

In the formal framework outlined in Section 1, this is tantamount to saying that an avoidance strategy is encoded in the function $f(\ldots)$ determining the evolution of the self-propelling force in Eq. 1; this strategy is not reducible to a simple social or mechanical force inducing radial repulsion. It remains to be checked whether the observed features (beyond the reach of simple social force models) can be captured by some of more elaborate lines of modelling, which provide a more realistic account of anticipating effects, in particular descriptions based on anticipated times to collision [7].

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